UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE

DEPARTMENT OF MATHEMATICS

MODULE	**MAT0CB2** ENGINEERING MULTIVARIABLE AND VECTOR CALCULUS
CAMPUS	**APK**
EXAM	**NOVEMBER 2015**

EXAMINER(S)	MRS C DUNCAN DR U KOUMBA
INTERNAL MODERATOR	MRS C MARAIS
DURATION	2 HOURS
MARKS	50

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 12

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL PAGES USED

Question 1 [3]

Given $f(x, y) = 1 + \sqrt{y - x^2}$.

(1.1) Determine the range of f. (1)

(1.2) Find and sketch the domain of f. (2)

Question 2 [4]

Given $g(x,y) = 100 - x^2 - y^2$.

(2.1) Give the name of the graph of g. (1)

(2.2) Graph g and plot the level curves corresponding to $k = 0$; 51; 75; 100. (3)

Question 3 [5]

(3.1) Suppose $F(x, y) = 0$ defines y implicitly as a differentiable function of x. If F is differentiable, then prove that

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

(3)

(3.2) If $x^2 + \sin y - 2y = 0$, detemine $\dfrac{dy}{dx}$. (2)

Question 4 [2]

If $x^3 + y^3 = 1 - 6xyz - z^3$, detemine $\dfrac{\partial z}{\partial y}$.

Question 5 [6]

Find the maximum and minimum distances from the point $(2, 1, -2)$ to the sphere centered at the origin with radius 1.

Question 6 [6]

Let Q be the solid bounded by $z = 3 - \sqrt{x^2 + y^2}$ and the planes $x = y$, $y = 0$ and $z = 0$. **Sketch** Q and set up triple integrals to represent the volume of Q as follows:

(i) In rectangular coordinates of the order $dzdydx$, AND

(ii) In cylindrical coordinates.

Question 7 [4]

Let E be the solid ellipsoid $x^2 + y^2 + 4z^2 = 9$ that lies in the first octant above the plane $z = 1$. Express the iterated integral

$$\iiint_E \frac{y}{z}\, dV$$

in spherical coordinates.

Question 8 [4]

Given a vector field $\mathbf{F}(x, y) = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$. Show that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P\,dx + Q\,dy + R\,dz$$

along a smooth curve C.

Question 9 [4]

Consider the following vector field $\mathbf{F} = \langle y - x,\, 2x - y \rangle$.

Determine the work done by $\mathbf{F}$ along the curve C, where C is the boundary of the region R, with R lying inside the semi-circle $y = -\sqrt{25 - x^2}$, but outside $y = -\sqrt{9 - x^2}$.

Question 10 [4]

If $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is a vector field on $\mathbb{R}^3$ and P, Q, and R have continuous second-order partial derivatives, then show that

$$\text{div curl } \mathbf{F} = 0.$$

Question 11 [8]

(11.1) Suppose that the plane region D, its boundary curve C, and the functions P and Q satisfy the hypothesis of Green's Theorem. Considering the vector field $\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$, prove the vector form of Green's Theorem

$$\oint_C \mathbf{F} \cdot \mathbf{n} \;\; ds \; = \iint\limits_D \operatorname{div} \mathbf{F}(x, y) \;\; dA$$

where $\mathbf{n}$(t) is the outward unit normal vector to C. (5)

(11.2) Using ONLY the result of (11.1), evaluate the integral

$$\oint_C xy\,dy \;-\; y^2\,dx$$

where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$. (3)